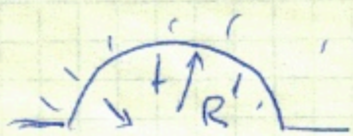
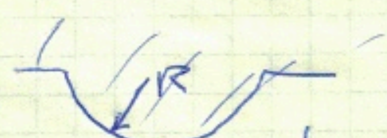
Analysis of the sintering process

Building a model for the sintering process has three fundamental parts as described below.

1. THERMODYNAMICS

1.1 Surface Tension creates pressure

$p_H = \frac{2\gamma_s}{R}$ (hydrostatic tension)

$p = -\frac{2\gamma_s}{R}$ (negative sign) (hydrostatic compression)

↓

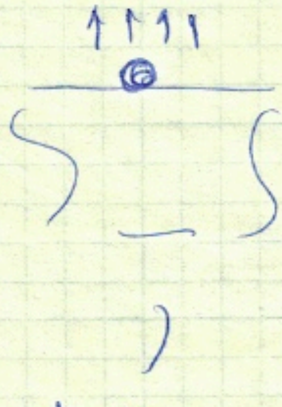
we deal with this as normal traction t_n

So then $t_n = \frac{2\gamma_s}{R}$

$t_n = -\frac{2\gamma_s}{R}$

1.2 The chemical potential of species depends on t_n (on the surface)

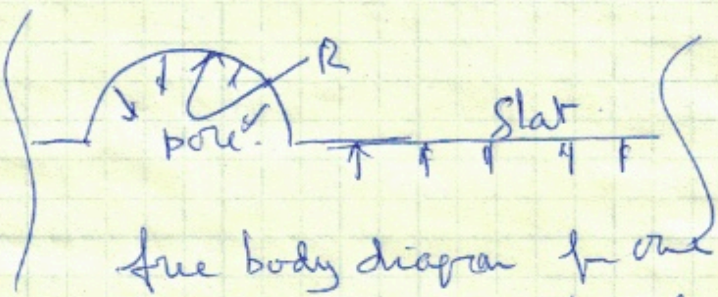
volume $= \Omega$

$$\Delta\mu = -t_n\Omega$$

Note the sign: the potential is lower on the right since atoms are flying away from it

1.3. Combine 1.1 & 1.2



free body diagram for one half of the grain boundary containing a pore.

$$\Delta\mu \simeq 2 t_n \Omega$$

(between flat & curved surfaces)

The factor of 2 appears because tension at the pore surface creates a compression on the flat part of the boundary. These are assumed to be equal although they are not (they depend on the relative area under tension (the pore surface) and compression the grain boundary

2. Kinetics (KINETICS)

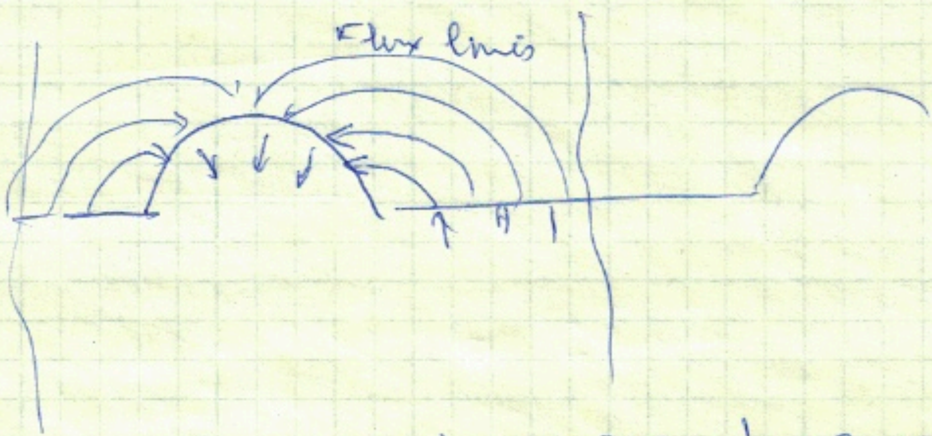
The gradient in chemical potential from the flat boundary into the pore will drive atom flux

$$J = \frac{D}{\Omega k_B T} \frac{d\mu}{dx} \sim \frac{\Delta\mu}{\text{distance}} = \frac{2 t_n \Omega}{p/2}$$

where p is the particle size.

3. GEOMETRY

3.1. Geometry defines the pathway for the flux. For example



Assumption: grain boundaries can be a source (depletion) or a sink for atoms (growth) — the mechanism being crystal etching or crystal growth.

3.2 $\phi = 2AJ$ number/sec.

(from each crystal of the boundary)

(the total atoms being transported per pore)

A: cross section for diffusion

$$A \approx \left(\frac{p}{2}\right)^2$$ for a 3D problem (spherical pores)

or $\approx \frac{p}{2} \times 1$ for a 2D problem where we consider a pore of unit length $\perp$ to the plane of the paper.

3.3 pore shrinkage

$$\frac{dV_{pore}}{dt} = \phi\Omega.$$

3.4 Shrinkage of the specimen.

This is related to the movement of the crystals that form the boundary towards each other. This displacement rate $\dot{U}$ is given by

$$\dot{U} = \frac{\phi}{\text{area of the boundary per pore}}.$$

3.5. Shrinkage strain rate.

$$\dot{\epsilon}_s = \frac{\dot{U}}{\text{distance between grain boundaries}}$$

$$= \frac{\dot{U}}{p}.$$

Now assemble all equations starts from the last one.